AMMUNITION PURCHASE DECISIONS IN THE FACE OF LONG LEAD TIMES AND USAGE UNCERTAINTY

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Ammunition purchase decisions are confounded by two significant factors: long lead times for suppliers to produce and deliver ammunition; and significant usage uncertainty. This puts a premium on careful planning to make sure that there are no shortages and, at the same time, that inventories are not excessively high. This paper presents a model for determining purchase requirements which trade-off these competing objectives.

Key words: *ammunition purchase, decision-making models, uncertainty, planning*

1. INTRODUCTION

Consider a defence planner who must determine how much of a particular nature of ammunition to purchase over the immediate short term. There are two critical factors which affect this decision. The first is that suppliers typically have long lead times. For example, the lead time for some Canadian Forces (CF) natures ("nature" means a specific type of ammunition) produced domestically is about two years. The second is that there is significant usage uncertainty. That is, there is significant year-to-year variation in usage. These two factors put a premium on careful planning to make sure that there are no shortages and, at the same time, that inventories are not excessively high. This paper presents a model for determining order sizes which trade-off these competing objectives.

Materials Requirements/ Resource Planning (MRP) and its successor, Enterprise Resource Management (ERP), are well developed technologies (particularly in the private sector) for matching manufacturing requirements and manufacturing schedules with final product demand requirements. However, these systems are only as good as the models underneath that inform them. For good models, we need to turn to the operational research literature on inventory management. The seminal reference is Petersen and Silver [1].

Most texts in management science and operations management devote whole chapters to techniques for efficient purchasing decisions. Examples are Anderson et al [2], Chase et al [3], and Nahmias [4]. That I am aware, there are few models which capture the essentials of the ammunition inventory problem I will describe below. Consequently, my purpose is to develop a simple model of ammunition purchase and inventory in the case where there is significant ammunition usage uncertainty and long lead times for purchase.

2. THE ORGANIZATIONAL PROBLEM

Most defence organizations purchase two kinds of rounds. One is used only in operations (stockpiles); the other is used in training. In this paper we focus on the purchase of training rounds since the problem of determining stockpiles is subject to a different set of performance characteristics. With stockpiles, the costs of running out of ammunition are significantly higher. In addition, training round inventories tend to be a much larger investment than stockpiles.

What is critical to this analysis is an understanding of the way ammunition supply and use are organized. For the CF, those who use the ammunition – the trainers – are a different part of the organization than those who are responsible for the purchase of ammunition. The ammunition requirement for training purposes is forecast at best a year in advance. Very often training schedules are not finalized until the very last moment. Moreover, the forecast of the ammunition requirement for individual courses is usually quite different than what is actually used for a variety of reasons. Very often, the actual number of soldiers taking a course is different than what was forecast, and, over the course, soldiers can fail the course before finishing it, thus reducing the ammunition required over the remainder of the course. All this to say, the CF has a difficult time estimating the ammunition for a single course and, what is more, there is substantial evidence that there is a systematic error in this estimate. For example, the forecast of the annual Army training requirement measured in dollars is usually substantially above what is actually used. In some years, the error is of the order of 30%. Suffice it to say, that a forecast of the requirement based on training plans is not reliable. Beyond that, training plans are forecast, at best, a year in advance which is a shorter time-frame than the lead-time for purchase. All this to say, in the absence of information to the contrary, the best predictor of a nature's requirement is past usage *aggregated over all training courses*. As it turns out, these usages have been fairly constant for most natures. The only time this would change would be where there is a material change to the force structure. For instance, if the Government of Canada approved an investment in a new infantry battalion, the training requirement for infantry natures would change substantially.

2.1. An Example

Suppose that aggregate usage of a particular nature over the past six years is as shown in the following table:

Table no.1.
Aggregate usage over 6 years

Year	Usage
y_{-1}	9,923
y_{-2}	10,424
y_{-3}	9,979
y_{-4}	10,804
y_{-5}	8,965
y_{-6}	11,085

In this table we use y−1 to refer to last year, y−2 for the year previous to that, and so on. Clearly these usages are variable and the planner must take this into account when determining how much to order. Suppose that the planner is looking forward in time and it is late in year y−1. His current inventory is 3,650 units, 10,000 units are due to be delivered at the start of year y1, and 11,000 units are due to be delivered at the start of year y_2. The planner must decide how much to order now for delivery at the start of year y_3. Moreover he would like to do this in such a way that there is a very low probability of a shortage in year y_3.

The first step is to characterize usage over years y_1, y_2, and y_3. With so little historical data, it is difficult to say what sort of probability distribution the usages are drawn from. However, on theoretical grounds, one could argue that the normal distribution is appropriate. The annual usage of a particular nature is the sum of a number of uncertain demands including individual training serials, collective training exercises, and other uses. Hence the Central Limit Theorem suggests that the annual usage, a sum of random in-year usages, ought to follow a normal distribution. Consequently we begin with the assumption that annual usage is drawn from a normal distribution with mean μ and standard deviation σ. In addition we assume that:

1. the distribution of year-to-year usages is stationary (each year's usage is drawn from the same normal distribution); and
2. the sequence of annual usages are independent random variables.

At a later point in the paper, we will discuss the implications if these assumptions are relaxed.

To get estimates for the parameters of the normal distribution, we could use standard maximum likelihood estimators (MLEs). In the general case, suppose that the history of usages over the past m years is

$$H = \{u-1, u-2, \dots, u-m\}. \quad (1)$$

Then the MLE for the mean is the sample average

$$\overline{u} = \frac{\sum u_{-i}}{m} \quad (2)$$

and for the standard deviation, it is *the sample standard deviation*:

$$s = \sqrt{\frac{\sum (u_{-i} - \overline{u})^2}{m-1}}. \quad (3)$$

For the dataset above,

$$\overline{u} = 10,197 \quad (4)$$

$$s = 755. \quad (5)$$

Returning to our example, let I_0 = 3, 650 be the current inventory, let x_1 = 10, 000 be the amount to be delivered at the start of year y_1, let x_2 = 11, 000 be the amount to be delivered at the start of year y_2, and let x_3 be the amount which is to be ordered for delivery at the start of year y_3. It is x_3 that the planner is trying to determine. Let U_i be the uncertain usage in year y_i for i = 1, 2, and 3. For year y3, we would like the inventory at the start of the year (including the order amount x3) to be sufficient so that there are no shortages. That is, we would like

$$\Pr(U_1 + U_2 + U_3 \leq I_0 + x_1 + x_2 + x_3) \quad (6)$$

to be quite high. Note that the left-hand side of the inequality is the aggregate use over the years y_1, y_2, and y_3; the right-hand side is what is available to use over this same period. Since U_i is a normal random variable for all i, $U_1 + U_2 + U_3$ is also normally distributed. The mean of this sum is 3μ and the standard deviation is $\sqrt{3}\sigma$. Using the MLEs derived above, we estimate that $U_1 + U_2 + U_3$ follows a normal distribution with mean

$$3\bar{u} = 30,590 \quad (7)$$

and standard deviation

$$\sqrt{3}s = 1,307. \quad (8)$$

Hence if $I_0+x_1+x_2+x_3$ is set at 1.645 standard deviations above the mean, 30,590, there is only a 5% chance of running out of ammunition. More generally,

$$I_0+x_1+x_2+x_3 = 3u + 1.645\sqrt{3}s \quad (9)$$

implying that

$$x_3 = 3u + 1.645\sqrt{3}s - I_0 - x_1 - x_2. \quad (10)$$

Substituting the numbers from our example, we have

$$x_3 = 30,590 + 1.645(1,307) - 3,650 - 10,000 - 11,000 = 8,090 \text{ rounds}. \quad (11)$$

Consequently, the planner should order approximately 8,100 rounds for delivery at the start of year y_3. At this order level there is about a 5% chance of running out in year y_3. Obviously a number of refinements are possible. Here are just a few:

1. The planner may be uncomfortable giving the same weigh to each usage in the data history. An obvious adjustment would be put more weight on the most recent observations. This is easily done using weighted sample estimators.

2. The planner may feel that the stationarity assumption on average usage is unwarranted. It is straightforward to adjust the model for drift upward or downward in the average usage.

3. The planner may feel that the usages in forward periods might be more (or less) variable than the usage history suggests. This is easily handled by allowing the planner to adjust the estimate of standard deviation.

4. The decision timings are likely to be slightly different than those we have modeled here. For instance it might be that the planner is trying to make a decision, say, half way through year y−1 for what to order for delivery at the start of year y3. The analysis for this case follows the same principles. We will have more to say about this below.

The planner may wish to alter the risk of stock-out depending on the nature he is considering. For instance, for the data above, suppose the planner wanted the probability of running out to be less than 1% rather than 5%. Then he would allow for 3 standard deviations above the mean:

$$\begin{aligned} x3 &= 3u + 3\sqrt{3}s - I0 - x1 - x2 \\ &= 30,590 + 3(1,307) - 3,650 - 10,000 - 11,000 \\ &= 9,862 \text{ rounds} \quad (12) \end{aligned}$$

With 3 standard deviations there is a less than a 1% chance of running out. The model is easily adjusted for this requirement. Note that moving to a 1% stockout risk increases the purchase quantity substantially. Our view is that 5% ought to be reasonable for most natures.

6. Other distributional assumptions are possible. Here we employed the normal distribution for good reasons. But a planner may prefer a distribution which ascribes higher variation such as the uniform distribution. Again, it is straightforward to make this kind of adjustment.

3. ASSESSING VOLUME DISCOUNTS

Very often, ammunition suppliers are prepared to offer price discounts if defence organizations are prepared to take delivery of a higher volume. Returning to our CF example, suppose a supplier is currently charging a price of p per unit for those units delivered in at the beginning of year y_3 but if the CF is prepared to purchase its y_3 and y_4 requirements at the beginning of year y_3, the all-units price is $p(1-\delta)$ where $\delta > 0$. The question is whether it is economic to make the higher volume purchase.

Taking the approach outlined above, suppose an analyst determines that the y_3 and y_4 requirements are x_3 and x_4 respectively. Then it will be cheaper to purchase all of this requirement at the beginning of year y_3 if

$$\frac{p(1-\delta)(x_3+x_4)}{(1+k)^2} < \frac{px_3}{(1+k)^2} + \frac{px_4}{(1+k)^3} \tag{13}$$

where k is an opportunity cost of capital. The left-hand side is the present value of the payment for the volume purchase; the right-hand side is the present value of two purchases, one at the start of year y_3 and the second at the start of year y_4. This inequality simplifies to the condition

$$\delta > \frac{k}{1+k}\left(\frac{x_4}{x_3+x_4}\right). \tag{14}$$

That is, the discount must exceed an adjusted purchase requirement fraction.

Here is an example. Suppose we take k to be 12% based on an assessment of the current government borrowing rate adjusted for risk and inventory costs. In addition, suppose that

$$\frac{x_4}{x_3+x_4} = .5. \tag{15}$$

Then we must have that

$$\delta > \frac{.12}{1+.12}(0.5) = .054. \tag{16}$$

That is, the price discount must be at least 5.4% for the volume purchase to be economic.

4. THE VALUE OF INFORMATION

The decision about how much to buy for year y_3 depends critically on the quality of the information the planner has. For instance it would be preferable to make a decision some time early in year y_1 rather than $y-1$ since the planner is likely to have better information about $u-1$ and I_0.

Having u−1 rather than an estimate means that his parameter estimates for the usage distribution will be more accurate. For the same reason, waiting to the end of the year to observe I_0 is preferable to estimating it at some point in year y−1. All things being equal, both of these factors allow the planner to order a smaller amount for a given level of stock-out risk. Hence, it is best to delay the purchase decision if such a delay allows the planner access to better data.

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